Calculus Year 13 (Level 8)

Summary 2

Bionomials, Functions

Factorials

$n! = n(n-1)(n-2)...2.1; n \in N$ is the number of ways n objects can be arranged.

$0! \equiv 1$

Simplify $\frac{73!}{71!} = \frac{73 \times 72 \times 71!}{71!} = 73 \times 72 = 5256$

Combinations

${}^nC_r$ is number of ways *r* objects can be selected from a total of *n* objects

${}^nC_r = \frac{n!}{r! \times (n-r)!}$ where ${}^nC_0 = 1$ and ${}^nC_n = 1$

Binomial Theorem

${}^nC_r$ are also coefficients in binomial expansions (and arranged in **Pascal's triangle**)

$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + ... + {}^nC_r x^{n-r} y^r + ... + {}^nC_n y^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} y^r$

Hence the (r+1)st term is $T_{r+1} = {}^nC_r x^{n-r} y^r$

Use this to calculate any term e.g. find term with x^8 in $\left(x^2 + \frac{1}{x}\right)^{10}$

${}^{10}C_r \left(x^2\right)^{10-r} \left(\frac{1}{x}\right)^r = {}^{10}C_r x^{20-2r} x^{-r} = {}^{10}C_r x^{20-3r}$ Hence $20 - 3r = 8$ and $r = 4$

So the term is ${}^{10}C_4 x^8 = 210x^8$

Functions

The function $y = f(x)$ **maps** the set of *x* **onto** the set of *y*.

The set of *x* is the **domain**; the set of *y* is the **range** of the function

Any value of x can only have one value of y associated with it, otherwise $f(x)$ is not a function

Notiations: $4x+5$ $y = 4x+5$ $f(x) = 4x+5$

$f: x \mapsto 4x+5$ (mapping) $\{(x,y): y = 4x+5\}$ (set builder)

INVERSE FUNCTION swap *x* and *y* in $y = f(x)$

Ex: $y = f(x) = x+6$ $f^{-1} \to x = y+6$ or $y = x-6$ Hence $f^{-1}(x) = x-6$

COMPOSITE FUNCTION $f \circ f^{-1} \equiv x$

Ex: substitute $y = x-6$ for x in $y = x+6$ i.e. $y = (x-6)+6 = x$