Ngā Whetū Resources

Calculus Year 13 (Level 8)

Summary 8

Parameters

A function $y = f(x)$ in "Parametric" form is written as $x = x(t);\ y = y(t)$ where t is the parameter. Eliminate t to obtain the "Cartesian" form $y = f(x)$.

Ex. $x = 3t \quad y = t^2$ hence $t = \frac{x}{3}$ and $y = \left(\frac{x}{3}\right)^2 = \frac{1}{9}x^2$

Differentiation of Parametric form

First differentiate with respect to the parameter: $\frac{dx}{dt} = x'(t);\ \frac{dy}{dt} = y'(t)$ then

$$y'(x) = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}$$

Tangent to a curve

Ex. $x = 2t^2 + 1;\ y = t^3 - 1$

a) Find the tangent in the point $t = 2$

First $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3t^2}{4t} = \frac{3}{4}t$

For $t = 2$ the gradient is $\frac{3}{4} \times 2 = \frac{3}{2}$ and $x = 2 \times 2^2 + 1 = 9;\ y = 2^3 - 1 = 7$

The tangent to the curve now is: $(y - y_1) = m(x - x_1) = (y - 7) = \frac{3}{2}(x - 9)$ or

$3x - 2y - 13 = 0$

b) Find the tangent in point $(3,0)$

Solve t from simultaneous equations: $3 = 2t^2 + 1;\ 0 = t^3 - 1$ giving: $t = \sqrt{1} = \pm 1$ in second equation gives $(\pm 1)^3 = 1$ hence $t = 1$

Now evaluate the gradient and x and y as above to find the gradient.

Normal to a curve

Same scheme as above but the gradient of the line is now not $\frac{dy}{dx}$ but $-\frac{1}{\frac{dy}{dx}} = -\frac{dx}{dy}$

Practice with exercises in the book (17.2)

Conics in Parametric form

Parabola	$y^2 = 4ax$	$x = at^2;\ y = 2at$	parameter t
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$x = a\cos\theta;\ y = b\sin\theta$	parameter θ
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$x = a\sec\theta;\ y = b\tan\theta$	parameter θ