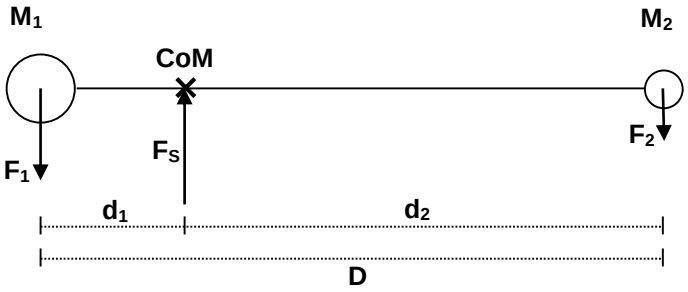
Physics Year 13 (NCEA Level 3)

Centre of Mass

The Centre of Mass (CoM), also called Centre of Gravity, is the point for an object where there occurs no rotation when a force is applied at that point.

Look at a see-saw:



Vertical equilibrium means that $F_S = F_1 + F_2$ but in addition equilibrium demands that the torque $F_1 \times d_1$ is equal to and in opposite direction of $F_2 \times d_2$ Hence $F_1 \times d_1 = F_2 \times d_2$. This only happens with respect to the CoM, thus the equation $F_1 \times d_1 = F_2 \times d_2$ is the actual condition to find the CoM.

With $F = Mg$ this condition can also be written as $M_1 \times g \times d_1 = M_2 \times g \times d_2$ or

$$\boxed{M_1 \times d_1 = M_2 \times d_2}.$$

So the location of the CoM depends on the ratio of the two masses. With a bit of algebra this condition can be re-arranged to

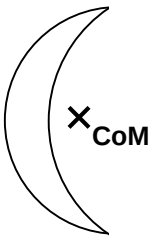
$$d_1 = \frac{M_2}{M_1 + M_2} \times D \text{ as well as } d_2 = \frac{M_1}{M_1 + M_2} \times D$$

These two expressions are handy to immediately calculate the location of the CoM from the two masses and their mutual distance D.

Assignment

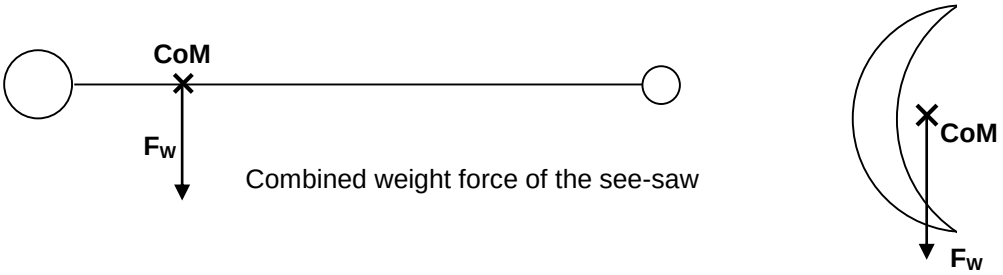
The Mass of the Earth is 5.976×10^{24} kg and that of the Moon is 7.349×10^{22} and their mutual distance is 3.844×10^{8} m. Calculate how far the joint CoM of the Earth-Moon system is away from the centre of the Earth. If you know that the radius of the Earth is 6.378×10^{6} m, what can you conclude about this joint CoM?

In a zero-gravity situation like during a Space Shuttle flight, when an astronaut pushes against an object floating in front of her and the force is applied exactly in line with the CoM of the object, there will be no rotation and the object will only be translated away from her.
Pushing at any other point will create a rotation of the object as well as a translation.



A CoM does not need to be inside the object. Think e.g. about the crescent shape as depicted here. The CoM is definetely outside the object.

The CoM is not a material point; it is only a concept which makes it easy to study equilibrium of forces acting on objects.
A weight force acting on an object is the sum of many small forces acting on small components of the object. It is much easier to combine these small forces into one weight force which is attached at the CoM of the object.



Motion
The CoM is also convenient when studying the motion of an object.