Rotational Motion and Energy
And Excersises
(Workings)

2. Radian as Angular measure

Exercises

a. Convert these angles to radians. Leave π in your answer:
90°, 60°, 120°, 270°, 2 revs, 1.5 revs
π/2; π/3; 2π/3; 3π/2; 4π; 3π.

b. Convert these angular speeds into rad s^{-1}:
2 revs per second, 4 revs in 0.2 s, 10 Hz, 600 RPM, time period $T = 5$ ms
4 π; 8π/0.2=40π; 10x2π=20π; 600/60x2π=20π; 2π/0.005=400π.

c. A sling consists of a stone spinning at the end of a string at 2.6 times per second around a circle with a radius of 1.5 m.
   i. Calculate the stone's angular velocity
   $\omega = 2\pi f = 2.6 \times 2\pi = 5.2\pi$ rads^{-1}

   ii. Calculate the stone's linear velocity
   $v = r\omega = 1.5 \times 5.2\pi = 7.8\pi$ ms^{-1}

d. Calculate the linear velocity of the edge of a 25 cm diameter grinding wheel spinning at 3600 RPM.
$\omega = \frac{3600}{60} \times 2\pi = 120\pi.$ $V = r\omega = 0.25 \times 120\pi = 30\pi$ ms^{-1}

e. A 25 cm radius wheel is set rolling along the ground at 0.65 ms^{-1}.
   i. Calculate its angular velocity
   $v = r\omega \rightarrow \omega = \frac{v}{r} = \frac{0.65}{0.25} = 2.6$ rads^{-1}

   ii. It rolls 2.8 m along the ground before stopping. Calculate how many radians it rotates through
   $d = r\theta \rightarrow \theta = \frac{d}{r} = \frac{2.8}{0.25} = 11.2 \text{ or } 11$ rad

   iii. How long does its journey take?
   $\theta = \omega t \rightarrow t = \frac{\theta}{\omega} = \frac{11.2}{2.6} = 4.3$ s

3. Rotational Kinematics

Exercises

a. A bike wheel accelerates from an angular speed of 10 rad s-1 to 210 rad s-1 over a time of 5.0 s.

i. Calculate the wheel's angular acceleration

$\omega_f = \omega_i + \alpha t \rightarrow \alpha = \frac{\omega_f - \omega_i}{t} = \frac{210-10}{5.0} = 40$ rads^{-2}

ii. Calculate how many radians it rotates

$\omega_f^2 = \omega_i^2 + 2\alpha\theta \rightarrow \theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{210^2 - 10^2}{2\times 40} = 550$ rad

iii. Convert the radians to the number of revolutions

$\frac{550}{2\pi} = 87.535$ or 88 rev.

iv. How long does it take to reach a speed of 90 rad s-1?

$\omega_f = \omega_i + \alpha t \rightarrow t = \frac{\omega_f - \omega_i}{\alpha} = \frac{90-10}{40} = 2.0$ s

v. What is the angular velocity when it has rotated through 200 rad?

$\omega_f^2 = \omega_i^2 + 2\alpha\theta \rightarrow \omega_f = \sqrt{10^2 + 2\times 40 \times 200} = 126.88$ or 130 rads^{-1}

b. A turntable slows down from 8.0 rad s^{-1} to 4.0 rad s^{-1}. During this time it rotates exactly seven times.

i. Calculate the angular displacement during this time

$\theta = 7 \times 2\pi = 14\pi$ rad

ii. Calculate the angular acceleration

$\omega_f^2 = \omega_i^2 + 2\alpha\theta \rightarrow \alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{4.0^2 - 8.0^2}{2\times 14\pi} = -0.5457$ or -0.55 rads^{-2}

iii. Calculate the time is takes

$\omega_f = \omega_i + \alpha t \rightarrow t = \frac{\omega_f - \omega_i}{\alpha} = \frac{4.0-8.0}{-0.5457} = 7.33$ or 7.3 s

iv. Calculate how many turns it has made after 5 s.

$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = 8.0 \times 5 - \frac{1}{2} \times 0.5457 \times 5^2 = 33.125$ rad

$\frac{33.125}{2\pi} = 5.272$ or 5.3 turns.

c. A car with 32 cm radius wheels is travelling at 16 ms^{-1}.

i. Calculate the angular speed of the wheels

$v = r\omega \rightarrow \omega = \frac{v}{r} = \frac{16}{0.32} = 50$ rads^{-1}

ii. The car slows down to a stop. AS this happens, the wheels rotate 25 times. Calculate the wheel's angular displacement

$\theta = 2\pi \times 25 = 50\pi$ rad

iii. Calculate the wheels angular acceleration

$\omega_f^2 = \omega_i^2 + 2\alpha\theta \rightarrow \alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{0^2 - 50^2}{2\times 50\pi} = -7.9577$ or -8.0 rads^{-2}

iv. Calculate the car's linear acceleration

$a = r\alpha = 0.32 \times -7.9577 = -2.5464$ or -2.5 ms^{-2}

v. Calculate the time it takes to stop

Linear: $v_f = v_i + at \rightarrow t = \frac{v_f - v_i}{a} = \frac{0-16}{2.5464} = 6.283$ s

Rotational: $\omega_f = \omega_i + \alpha t \rightarrow \ t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0-50}{-7.9577} = 6.283$ or 6.3 s

vi. How far does the car travel while braking?

Linear: $d = v_i t + \frac{1}{2}at^2 = 16 \times 6.283 - \frac{1}{2} \times 2.5464 \times 6.283^2 = 50.26$ or 50 m

Rotational: $\theta = \omega_i t + \frac{1}{2}\alpha t^2 = 50 \times 6.283 - \frac{1}{2} \times 7.9577 \times 6.283^2 = 157.086$

$d = r\theta = 0.32 \times 157.086 = 50.26$

Or use $\omega_f^2 = \omega_i^2 + 2\alpha\theta \rightarrow \ \theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{0^2 - 50^2}{2 \times -7.9577} = 157.08$ etc.

4. Rotational Dynamics

Exercises

a. A tangential force of 2.5 N acts on a turntable with a radius of 32 cm. The turntable takes 1.2 s to complete the first revolution from rest.

i. What is the angular displacement for one revolution?

2π rad

ii. Use an equation of motion to find the angular acceleration

$\theta = \omega_i t + \frac{1}{2}\alpha t^2 \rightarrow \ \alpha = \frac{\theta - \omega_i t}{\frac{1}{2}t^2} = \frac{2\pi - 0}{\frac{1}{2} \times 1.2^2} = 8.7266$ or 8.7 $rads^{-2}$

iii. Calculate the torque acting on the turntable

$\tau = Fr = 2.5 \times 0.32 = 0.80$ Nm

iv. What is the rotational inertia of the turntable?

$\tau = I\alpha \rightarrow I = \frac{\tau}{\alpha} = \frac{0.80}{8.7266} = 0.09167$ or 0.092 kgm^2

b. A cord is wrapped around the axle of a wheel. A 6.5 kg mass on the cord accelerates the wheel from rest. The angular acceleration is 1.8 rad s^{-2}. The radius of the axle is 0.12 m. (Use g=10 ms^{-2})

i. Calculate the tension force in the cord

$\boldsymbol{F = mg = 6.5 \times 10 = 65}$ N

ii. Calculate the torque acting on the wheel

$\tau = Fr = 65 \times 0.12 = 7.8$ Nm

iii. Calculate the rotational inertia of the wheel

$\tau = I\alpha \rightarrow I = \frac{\tau}{\alpha} = \frac{7.8}{1.8} = 4.333$ or 4.3 kgm^2

iv. The torque is applied for 5 s. Calculate the angle turned through in this time

$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} \times 1.8 \times 5^2 = 22.5$ or 23 rad

c. A student uses a rowing machine and must pull on a chain wrapped around the axle of a flywheel, causing the flywheel to rotate. During the first pull the flywheel is uniformly accelerated from rest. This acceleration occurs over a distance (along the chain) of 0.48 m and takes a time of 0.85 s. The radius of the axle is 1.2 cm and the radius of the flywheel is 25 cm.

i. Show that the angle the flywheel turns through is 40 rad.

$d = r\theta \rightarrow \theta = \frac{d}{r} = \frac{0.48}{0.012} = 40$ rad

ii. Calculate the angular acceleration

$\theta = \omega_i t + \frac{1}{2}\alpha t^2 \rightarrow \alpha = \frac{\theta - \omega_i t}{\frac{1}{2}t^2} = \frac{40-0}{\frac{1}{2}\times 0.85^2} = 110.7266$ or 110 rads^{-2}

iii. The average force that is applied to the chain is 185 N. Calculate the torque applied to the axle

$\tau = Fr = 185 \times 0.012 = 2.22$ or 2.2 Nm

iv. Calculate the flywheel's rotational inertia.

$\tau = I\alpha \rightarrow I = \frac{\tau}{\alpha} = \frac{2.22}{110.7266} = 0.02005$ or 0.020 kgm^2

v. If an axle is used with a larger radius, will it be harder or easier to achieve the same angular acceleration? Explain.

$\tau = I\alpha \text{ and } \tau = Fr\text{: same } \alpha \rightarrow \text{same } \tau \rightarrow \text{when } r \text{ increases } F \text{ gets smaller}$

Hence it will be easier.

But $d = r\theta \rightarrow$ pull over a longer distance to achieve same rotation

5. Rotational Inertia and Mass distribution

Exercises

a. Using the equation $\tau = I\alpha$ show that the unit for Rotational Inertia is kgm^2.

$\tau = I\alpha \rightarrow I = \frac{\tau}{\alpha} \rightarrow$ Unit of I is $\frac{\text{Nm}}{\text{rads}^{-2}}$

$F = ma \rightarrow$ unit N equals unit kgms^{-2}

Combined: unit of I is $\frac{\text{kgm}^2\text{s}^{-2}}{\text{rad s}^{-2}} = \text{kgm}^2$

b. A dumbbell consists of two 1.5 kg masses that are 1.2 m apart. Calculate its rotational inertia with respect to its centre.

$I = \sum mr^2 = 1.5 \times 0.60^2 + 1.5 \times 0.60^2 = 1.08$ or 1.1 kgm^2

c. An ice skater stands straight up and has a certain rotational inertia about her vertical axis. She now spreads her arms wide. What happens to her rotational inertia? Explain.

Mass (of the arms) is moved away from the central axis. Hence I increases.