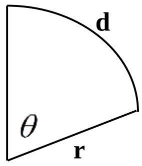
Physics Year 13 (NCEA Level 3)

Rotational Motion and Energy

Summary

Angular form of Equations of Motion



Linear displacement along a circle arc ***d*** (m) is compared to **Angular displacement** θ (rad). These are related by $\boxed{d = r\theta}$.

(Note that radians are simply defined this way: the circumference is $2\pi r$ and the angle of a full circle is 2π radians).

Similarly we can compare linear velocity ***v*** (ms^{-1}) to **angular velocity** ω ($rads^{-1}$) with the relationship $\boxed{v = r\omega}$ and
linear acceleration a (ms^{-2}) to **angular acceleration** α ($rads^{-2}$) with the relationship $\boxed{a = r\alpha}$.

The linear equations of motion can now be "translated" into the rotational equations of motion:

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

Angular Frequency

For circular motion we defined frequency f as number of revolutions per second (unit s^{-1})
Here we also define **Angular frequency** ω as number of radians per second (unit $rads^{-1}$)
There are 2π radians in one revolution, therefore $\omega = 2\pi f = \dfrac{2\pi}{T}$ where T is the Period (s).
This Angular Frequency is the same as the angular velocity ω above
Compare Linear velocity $v = \dfrac{2\pi r}{T}$ with Angular velocity $\omega = \dfrac{2\pi}{T}$ (remember $v = r\omega$).

Torque and Rotational Inertia

Torque is "Angular Force" caused by a force couple or by a force *F* at a distance *r* from the leverage point: $\tau = Fr$ unit is Nm.
Torque is proportional to angular acceleration: $\tau = I\alpha$.
(compare to Newton's second law: $F = ma$)
I is the **Rotational Inertia**. Unit of I is kgm^2.
Mass is a single constant for any object; Rotational Inertia depends on the mass *distribution* over the object: $I = \sum mr^2$

Energy in Rotation

Work is Torque times Angular displacement: $W = \tau\theta$

Rotational Kinetic Energy: $E_{rot} = \frac{1}{2}I\omega^2$

Energy in rolling is sum of linear and rotational energy: $E_{rolling} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Angular Momentum

compare with $p = mv$: $L = I\omega$ (unit kgm^2s^{-1})

Angular Momentum is conserved when there is no external torque.

Examples: rotating ice skater, somersault, air masses in atmosphere of rotating earth. If I changes because mass distribution changes, angular velocity must also change to conserve Angular Momentum.

Angular Momentum of a point mass is simply the Linear Momentum times the radius: $L = mvr$

Remember the (confusing) units for:

Linear Momentum	$kgms^{-1}$
Angular Momentum	kgm^2s^{-1}
Rotational Inertia	kgm^2