

Tutorial Modern Physics

REFERENCES

Online Physics Lab: <http://dev.physicslab.org/>

HyperPhysics: <http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>

Physics 1 & Physics 2, David Sang, Ed. Cambridge University Press

1. PARTICLE NATURE OF LIGHT

One of the experiments that convinced 19th century physicists that light is a wave phenomenon is Young's double slit experiment, showing diffraction and interference. But towards the end of the 19th century, it had been experimentally observed that when ultraviolet light was shone on a negatively-charged electroscope, the electroscope discharged. Hence there was a flow of electrons caused by the UV light. But that in itself was not in contradiction with the wave model of light. However in 1905 Albert Einstein performed the **photo-electric experiment**, which demonstrated that light interacts with a metal in a way that can only be explained with a particle model of light.

When light shines on any metal surface, the metal can release electrons. If light were composed of waves, then eventually any wavelength of light should be able to build up enough energy to knock an electron free. However, Einstein had discovered that only certain wavelengths worked with each metal and that electrons were either emitted instantaneously, or never emitted. He had also noticed that shorter wavelengths worked better than longer wavelengths.

More specifically the most remarkable aspects of the photoelectric effect are:

- The electrons are emitted immediately; there is no time lag
- Increasing the intensity of the light increases the number of photoelectrons, but not their maximum kinetic energy
- If a certain wavelength of light does not cause the ejection of electrons, increasing the intensity will not make a difference
- The maximum kinetic energy of the electrons is dependent on the light frequency, not on the light intensity

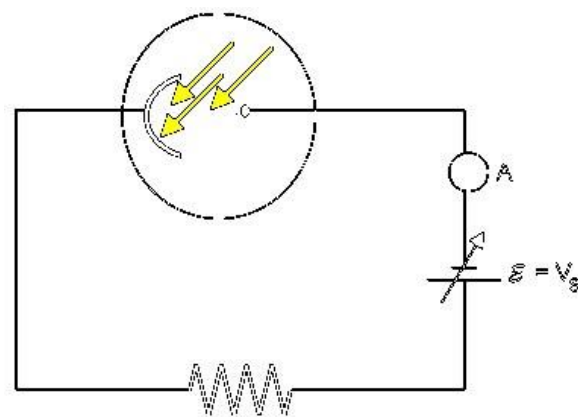


Image source: http://dev.physicslab.org/Document.aspx?doctype=3&filename=AtomicNuclear_PhotoelectricEffect.xml

This showed that whatever was knocking the electrons off had an energy proportional to the light frequency. The remarkable fact that the ejection energy was independent of the total energy of illumination showed that the interaction must be like that of a **particle** which gave all of its energy to an individual electron! This fits in well with Planck's hypothesis that light can exist only in discrete amounts of energy. In 1900 Max Planck had concluded that photon energy is dependent on frequency by

$$E=hf$$

Where E is the photon energy and h is Planck's constant (6.63×10^{-34} J sec). This photon energy explains the outcome of the experiment and shows that light has particle properties.

The Experiment

The electrons with kinetic energy E_k can be stopped from completing their flow across the photoelectric tube if there is a minimum **stopping potential** set-up to just impede their flow.

E_k relates to the Voltage by

$$E_k = V_s e$$

where e is the charge of an electron and V_s the stopping potential. So measuring the stopping voltage gives the kinetic energy of the electrons.

Einstein's equation for the photoelectric effect is

$$hf = \Phi + E_k$$

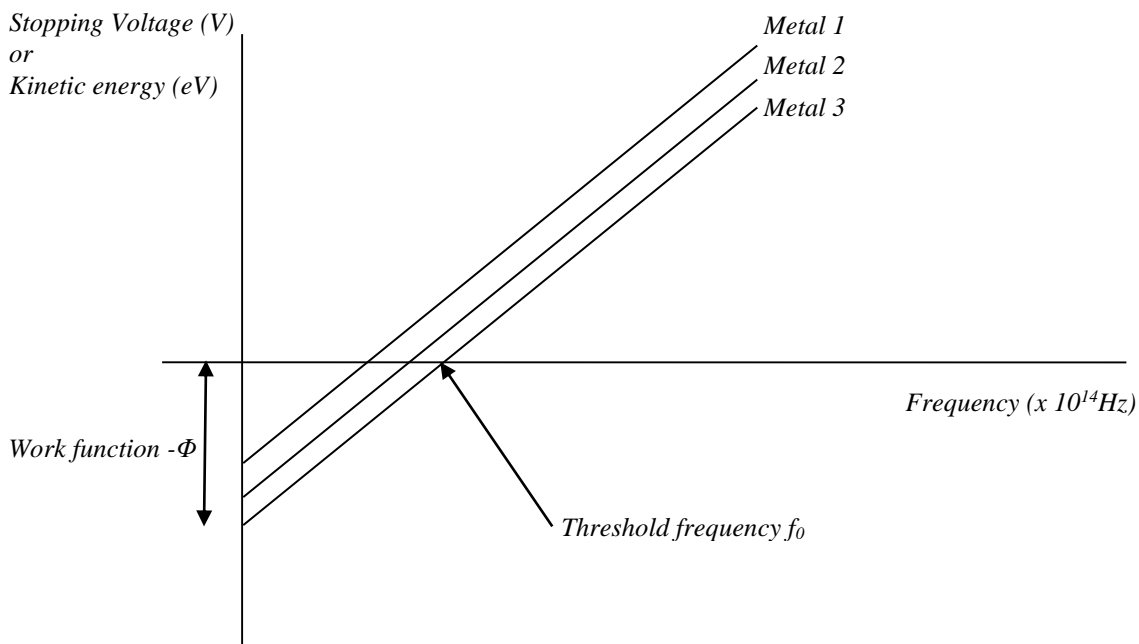
where hf is the energy of each incident photon, Φ is the amount of energy needed to release an electron from the metal, and E_k is the maximum kinetic energy of the escaping electrons.

Φ is called the **Work Function** and is specific to each metal. Since each photon can be absorbed by only one photoelectron, the energy of the photons directly affects the kinetic energy of the released photoelectrons. Basically this equation states conservation of energy.

Often the photoelectric equation is illustrated on a graph of E_k (or $V_s = E_k/e$) against frequency according to the re-arranged formula

$$E_k = hf - \Phi$$

This is a linear function of f and the slope of the line is ALWAYS equal to Planck's constant. The parallel lines on this graph only differ in their y-axis intercept ($-\Phi$) and their x-axis intercept (the threshold frequency). Each metal with a specific work function is represented by one line in the graph.



Examples of the value of the Work function for a number of metals
(from Handbook of Chemistry and Physics).

The threshold frequency f_0 is the value for which E_k is zero, hence

$$hf_0 = \Phi.$$

This is the minimum frequency the photons must have to release electrons from the metal. Photons of light with lesser frequency can never cause the photoelectric effect, no matter how intense the radiations is.

Albert Einstein received the Nobel Prize for Physics in 1921 for his discovery of the Law of the Photoelectric effect. His work clearly demonstrated the quantum nature of light and ended the controversy as to whether light had particle properties. This discovery, together with the work of other eminent scientists in the first half of the 20th century, formed the basis of **Quantum Mechanics**, defining the post-Newtonian era in Physics.

Wave – Particle duality

The dilemma now is that some phenomena involving light (diffraction, interference, polarisation, etc.) can only be explained with a wave model and other phenomena (photoelectric effect, spectral lines, etc.) can only be explained with the particle (photon) model. So both models are needed in physics. This is called the Wave – Particle Duality.

It has been shown that while light has particle properties, particles also have wave properties. Young's double-slit experiment can be carried out with a beam of electrons. This produces an interference pattern on the screen, just like it does with light. Measuring the distance between the maxima in the pattern allows us to calculate the wavelength of electrons.

Wavelength λ and frequency f are concepts in the wave model and e.g. photon energy E and momentum mv are concepts in the particle model. With Einstein's mass-energy equivalence $E = mc^2$ comparisons can be made. In this way deBroglie proposed in 1924 that a moving particle with linear momentum mv has a wavelength of $\lambda = h / mv$.

One important aspect of this duality is that both waves and particles are concepts that we know from the macroscopic (human scale) world and physicists use these concepts to model the world of sub-atomic, fundamental particles. This clearly is not really possible. However physicists make it acceptable by giving rules that tell us when to use which model. This as such has been very successful and allows modern physics to be a very successful extension of Newtonian physics. In the context of Quantum Mechanics "particles" are better viewed as energy "quanta", and waves as "probability waves".

Now practice with exam questions on the Photoelectric experiment.

Element	Work Function(eV)
Aluminium	4.08
Beryllium	5.0
Cadmium	4.07
Calcium	2.9
Carbon	4.81
Caesium	2.1
Cobalt	5.0
Copper	4.7
Gold	5.1
Iron	4.5
Lead	4.14
Magnesium	3.68
Mercury	4.5
Nickel	5.01
Niobium	4.3
Potassium	2.3
Platinum	6.35
Selenium	5.11
Silver	4.73
Sodium	2.28
Uranium	3.6
Zinc	4.3

2. ATOMIC STRUCTURE

The **ancient Greek** already proposed that all matter is made up of atoms (*átomos* means indivisible), but it was not before the 1850's that any suggestions were made about the structure of the atom.

Thomson suggested that the atom was made up of positively charged material with negatively charged lumps embedded in it. He discovered the electron (cathode rays) in 1897. With alpha-particles that were discovered from studying radioactivity, experiments could be made to find out more about the structure of the atom, because alpha-particles are considerably smaller than an atom. In 1911 **Rutherford** exposed thin gold foil to alpha-radiation and noted that most particles went straight through. Only a tiny fraction of the alpha-particles were deflected, sometimes they bounced straight back! Rutherford concluded that the atom must be mostly empty and have a heavy nucleus, that when hit, could bounce an alpha-particle back like a collision between snooker balls. He estimated the size of the nucleus to be about 10^{-14} m (some tenthousand times smaller than the atom itself). Later it was shown that the "collision" is due to the strong electromagnetic repulsion between the nucleus and the alpha-particle which are both positively charged.

Rutherford discovered the proton in 1919 and **Chadwick** discovered the neutron in 1932. These discoveries led to the "classical" model of an atom, in which the **nucleus** consists of positively charged **protons** and neutral **neutrons**, surrounded by a cloud of negative **electrons** that have a mass of about one tenthousandth of a proton.

Nuclei (plural of nucleus) consist of **nucleons** (protons and neutrons). Hydrogen is the most abundant element in the Universe and is the simplest atom, consisting only of one proton and one electron. All other elements have a nucleus that consists of a combination of protons and neutrons. A neutral atom has a number of electrons that is equal to the number of protons.

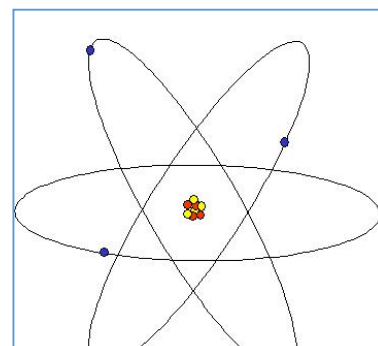


Image source:
www.detoxifynow.com/water_and_tao.html

Towards the heavier natural elements the number of neutrons becomes larger than the number of protons. Gold has 118 protons and 79 neutrons, and Uranium has 143 protons and 92 neutrons. An element is uniquely determined by the number of protons (**atomic number**). All atoms can have various **isotopes** in which the number of neutrons varies. The total number of nucleons is called the nucleon number (or **mass number**) of the atom. In the **Periodic Table of Elements** atoms are ordered by increasing atomic number and in rows according to their chemical properties.

(More about the nucleus and nuclear equations in the next chapter)

The Bohr model

When white light is directed through a prism or diffraction grating, we see a **spectrum** of many colours as in a rainbow. When we analyse the spectrum of a hot gas, we see that only certain colours are present as thin lines. This is an **emission spectrum**. When white light passes through a cool gas we see an **absorption spectrum** that has dark lines where specific colours are absent. It is clear from these observations that an element can only emit or absorb light of specific wavelengths. How can that be explained?

Niels Bohr proposed in 1913 that electrons in atoms can only exist at specific energy levels, and that when an atom becomes excited (e.g. in a hot gas) electrons jump up to a higher energy level and when these electrons subsequently fall down to a previous level, they emit light of a wavelength that represents a specific photon energy equal to the difference between the two energy levels; hence the emission spectrum. An absorption spectrum results from electrons absorbing specific amounts of energy from light that shines through a gas by jumping up to higher energy levels. Thus the light with photon energy equal to those energy differences is absent in the spectrum. Bohr's conclusion was that electron energy is **quantised**, which formed one of the most important results on which Quantum Mechanics is based.

The Bohr model of the atom is a simple model (based on classical mechanics) that works well for Hydrogen, but in modern Quantum Mechanics, more complex models for the electron configuration in atoms must be used (valence shell model). The Bohr model became very popular when **Rydberg** constructed a formula that accurately predicts the various spectral lines of Hydrogen. Rydberg's formula does not explain why electrons can only exist at these specific energy levels. But Bohr proposed that the energy levels are a consequence of the **quantisation of angular momentum** of the orbiting electrons.

The Rydberg formula

The energy of a photon emitted when an electron in a Hydrogen atom falls down from energy level n_i to level n_f (which are integers) is given by

$$E = E_i - E_f = R_E \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Because the photon energy relates to wavelength as

$$E = hf = \frac{hc}{\lambda}$$

the wavelength of the emitted photon is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where

$$R = \frac{R_E}{hc}$$

is the Rydberg constant ($1.097 \times 10^7 \text{ m}^{-1}$).

The New Zealand curriculum notation of the Rydberg formula is

$$\frac{1}{\lambda} = R \left(\frac{1}{S^2} - \frac{1}{L^2} \right)$$

S is the **final** energy level and **L** is the **initial** energy level of the electron.

Although Hydrogen has only one electron, there are many different energy transitions it can make. Therefore there are many lines in the emission and absorption spectrum of Hydrogen.

Different values of S (the final level) give different series of wavelengths called after their discoverers:

- $S=1$ Lyman series (Ultra Violet)
- $S=2$ Balmer series (visible light)
- $S=3$ Paschen series (Infra Red)

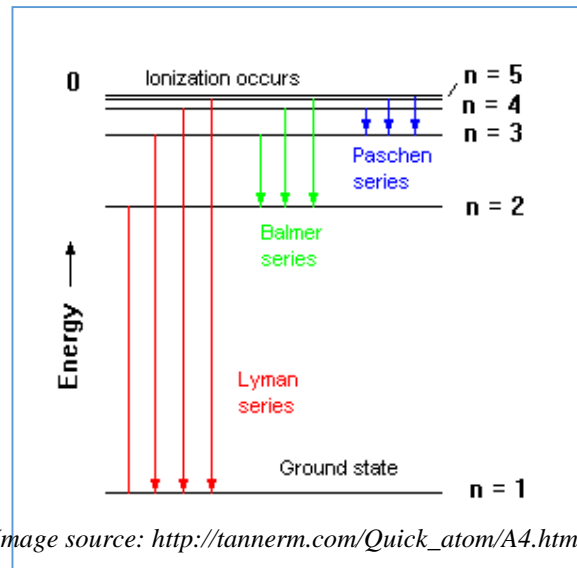
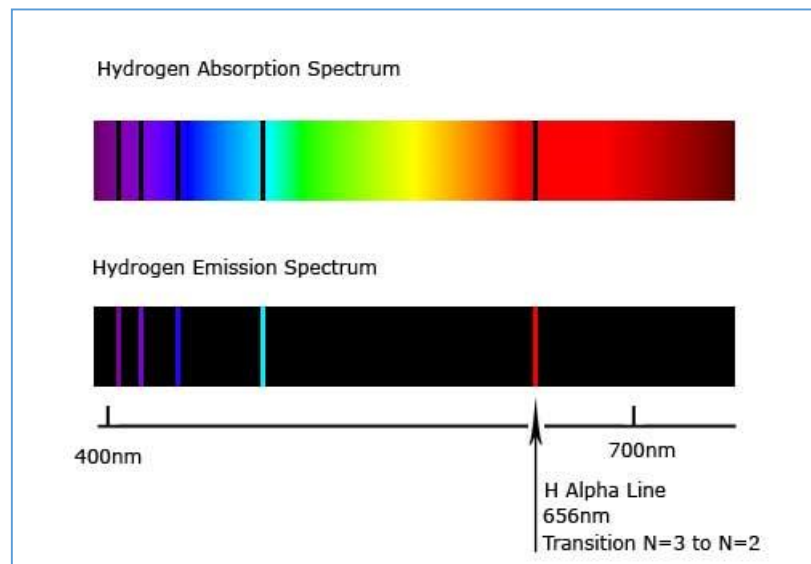


Image source: http://tannerm.com/Quick_atom/A4.htm

Example

The Balmer series of Hydrogen lines in the visible part of the spectrum
 Image source:



www.astronomyknowhow.com/hydrogen-alpha.htm

In this example the Balmer **emission** line from level 3 is labeled. Its wavelength can be calculated from Rydberg's formula as follows:

$$\frac{1}{\lambda} = R \left(\frac{1}{S^2} - \frac{1}{L^2} \right) = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 1.5236 \times 10^6$$

Hence

$$\lambda = 6.56 \times 10^{-7} = 656 \times 10^{-9} = 656 \text{ nm.}$$

For the **absorption** line we would use the jump up from $n=2$ to $n=3$ which gives the same answer with a negative sign, i.e. it is absorbed energy.

Ionisation

When an electron is entirely removed from the atom, the value for S becomes ∞ . Thus

$$\frac{1}{\lambda} = R \left(\frac{1}{\infty^2} - \frac{1}{L^2} \right) = -R \frac{1}{L^2}$$

With

$$E = hf = \frac{hc}{\lambda}$$

this can be written as

$$E = -\frac{hcR}{L^2}$$

and (inconsistently) in the New Zealand curriculum notation

$$E_n = -\frac{hcR}{n^2}$$

Note that h , c and R are all constants.

This formula can be used to calculate the “absolute” electron energy of any level (which is always negative in comparison to the highest (∞) level that is defined as $E_\infty \equiv 0$).

E.g. the energy at level 2 is:

$$E_n = -\frac{hcR}{n^2} = -\frac{6.63 \times 10^{-34} \times 3 \times 10^8 \times 1.097 \times 10^7}{2^2} = -5.455 \times 10^{-19} \text{ J}$$

To calculate this in the (for Atomic Physics more usual) **electronVolt** (eV) unit we divide by the charge of an electron

$$-\frac{5.455 \times 10^{-19}}{1.60 \times 10^{-19}} = -3.41 \text{ eV}$$

Note that the difference between adjacent energies becomes progressively less when going up.

$n = \infty$	-----	$E = 0$
$n = 4$	=====	$E = -0.85 \text{ eV}$
$n = 3$	=====	$E = -1.51 \text{ eV}$
$n = 2$	=====	$E = -3.40 \text{ eV}$
$n = 1$	=====	$E = -13.6 \text{ eV}$

Now practice with exam questions on the Atomic Structure.

Image source:

<http://spiff.rit.edu/classes/phys301/lectures/comp/comp.html>

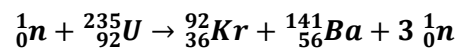
3. NUCLEAR PHYSICS

Nuclear equations

Nuclei are represented by a symbol for the element with the mass number (A) as a top index and the atomic number (Z) as bottom index.

$${}^A_ZX ; \text{examples } {}^{16}_8\text{O} (\text{Oxygen}), {}^{197}_{79}\text{Au} (\text{Gold})$$

A nuclear reaction is a process where a large nucleus splits in two or more smaller nuclei (nuclear fission) or where two small nuclei fuse to form a larger nucleus (nuclear fusion). Such a reaction can be represented in a nuclear equation. An example:



In this example a neutron hits a ${}^{235}\text{U}$ nucleus which results in a ${}^{92}\text{Kr}$ and a ${}^{141}\text{Ba}$ nucleus together with three new neutrons. There are two rules for such equations, based on conservation of mass and charge.

1. The sum of the top indices (mass number = number of nuclei) left and right of the arrow must be equal.
2. The sum of the bottom indices (atomic number = number of protons or units of charge) left and right of the arrow must be equal.

Verify that these rules are met in the equation above.

Radioactivity

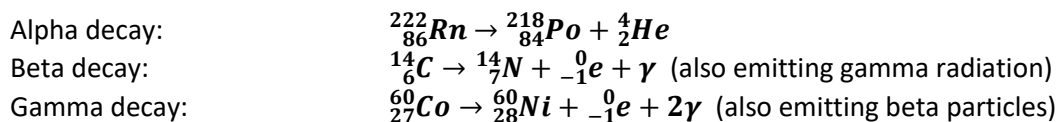
The naturally occurring elements are those with atomic number up to 92 (Uranium). The ones beyond $Z=83$ are unstable and gradually decay. This radioactive decay is a natural nuclear process in which larger nuclei change the ratio between neutrons and protons in the nucleus to obtain a more stable configuration. In most cases this involves the change of element because the proton number changes. This decay is accompanied by the emission of radiation and often also energy in the form of heat. There are three forms of nuclear radiation: alpha, beta and gamma radiation.

Alpha particles are helium nuclei ${}^4_2\text{He}$. They have a mass of 4 units¹ and charge of +2e

Beta particles are electrons ${}^0_{-1}e$ that have very small mass (1/1840 units) and charge of -1.

Gamma radiation is EM-radiation with the largest photon energies. This form of radiation can only originate from nuclear processes and not from electron transitions such as lower energy photons.

Examples of radioactive decay



¹ Masses at nuclear scale expressed in kg give very small numbers. More commonly masses of nuclear particles are given in Unified Atomic Mass (u) where the mass of the ${}^{12}_6\text{C}$ (carbon) nucleus is 12 u . Likewise the unit of charge is e , the charge of an electron.

Penetrating power

These forms of radiation affect the matter they pass through and cause ionisation (removal of electrons). The larger alpha particles have a strong ionisation effect and have therefore the smallest penetrating power. They soon lose their energy and are absorbed. Gamma radiation has the least ionisation power and therefore the highest penetrating power.

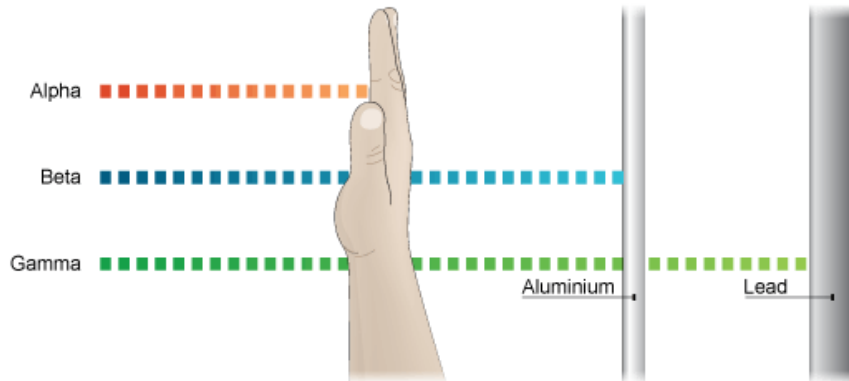


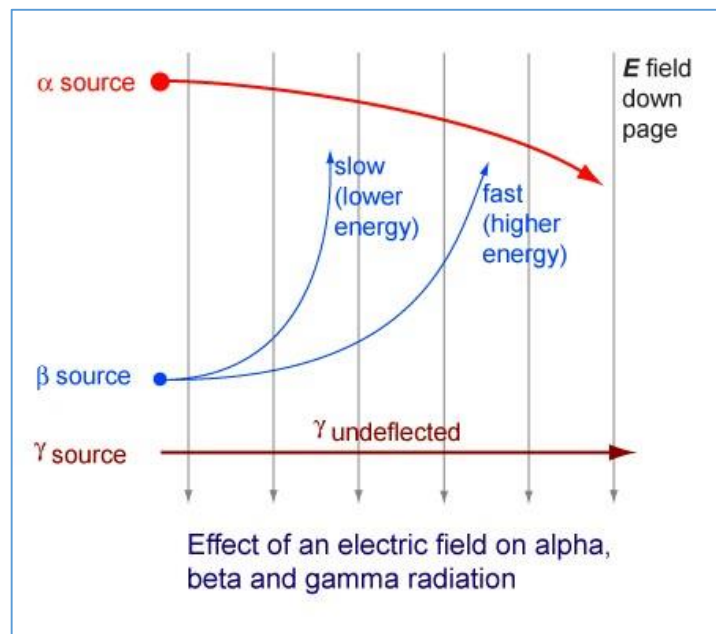
Image source:

www.bbc.co.uk/scotland/learning/bitesize/standard/physics/health_physics/nuclear_radiation_rev3.shtml

Deflection

Electric - and magnetic fields will deflect alpha and beta radiation due to the charge of the particles. Gamma radiation will not be deflected by either of these fields. The amount and direction of deflection depend on the **charge**, the **mass number** and the **speed** of the particles.

Image source:



http://outreach.atnf.csiro.au/education/senior/cosmicengine/sun_nuclear.html

Radiation hazards

The ionising effect of radiation is a hazard for living organisms. Strong radiation can kill cells which affect the tissue as a whole. Less intense radiation can still damage DNA or break up other molecules and thus interfere with cell functions.

Because alpha and beta particles are relatively non-penetrating, external exposure to them causes only localized damage, e.g. radiation burns to the skin. Gamma rays are more penetrating and cause damage throughout the body (e.g. radiation sickness) rather than burns. In addition to external radiation, exposure can also be internal due to ingested or inhaled radioactive substances.

Because of this hazardous effect, these types of radiation can also be applied in a controlled and beneficial way, e.g. in medicine to kill cancer cells and for sterilisation of food or equipment.

Half life

Radioactive decay is inherently a random process and cannot be predicted at the level of individual nuclei. But it can be approached statistically over large samples. Mathematically it then is a process of **exponential** decay.

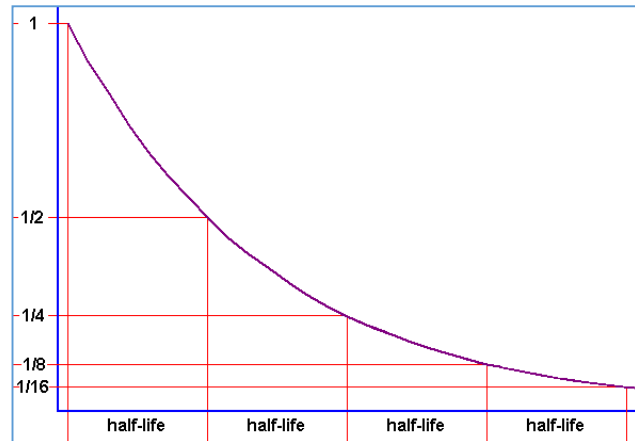
Image source: www.uwgb.edu/dutchs/EarthSC202Notes/earthage.htm

The **half-life** of a radioactive element is the average time for half of the nuclei in a sample to decay. It then takes the same amount of time for half of the remainder to decay and a third half-life for half of the new remainder to decay, and so on. The half-life of an element is a fixed value for that element and cannot be altered in any way. Different elements have different half-lives.

Examples:

Uranium	4.5×10^9 years
Polonium	3×10^{-7} s

This phenomenon is used e.g. to date geological samples of different materials. A famous example is the **Radio Carbon method** that can be applied to organic materials, which utilises the decay of the $^{14}_6\text{C}$ isotope that has a half-life of 5,500 years.



Mass–Energy equivalence

We are familiar with the classical law of conservation of energy. But in the Theory of Relativity Einstein showed that energy and mass are equivalent through the famous relationship

$$E = mc^2$$

Therefore in modern physics we must apply the law of **conservation of mass-energy**. It does not matter (no pun) if some mass is converted into energy or the other way around. But at all times the total mass-energy in a system must be conserved.

According to the Theory of Relativity the mass of an object is dependent on its speed. A fast moving object has larger mass than when it is stationary. For moving cars and flying aeroplanes this mass increase is negligible but at the subatomic level it must be taken into account. Therefore the mass of subatomic particles is expressed as **rest mass**, which is the mass when the particles are stationary. Rest mass is determined to a high number of significant figures because the small differences between the masses are important for mass-energy calculations.

Binding energy

If we study the mass of a nucleus as a whole and compare this with the sum of the masses of the individual nucleons we see that the latter is larger (see graph). We had to do work to pull the system of particles apart. This energy has been converted into mass: the separate nucleons have more mass than the nucleus itself. This difference in energy is called the **binding energy**.

If a nucleus is formed from the individual nucleons energy is released. That same amount of energy is required to pull the nucleons apart against the nuclear forces that hold them together. The more energy is involved in this, the more stable the nucleus is.



Image source (edited)

http://nobelprize.org/nobel_prizes/physics/articles/fusion/index.html

Example:

The mass of the $^{12}_6\text{C}$ nucleus is $19.926483 \times 10^{-27}$ kg whereas the six protons and the six neutrons together have a mass of $6 \times 1.672623 \times 10^{-27} + 6 \times 1.674929 \times 10^{-27} = 20.085312 \times 10^{-27}$ kg
The difference is $20.085312 \times 10^{-27} - 19.926483 \times 10^{-27} = 0.158829 \times 10^{-27}$ kg.

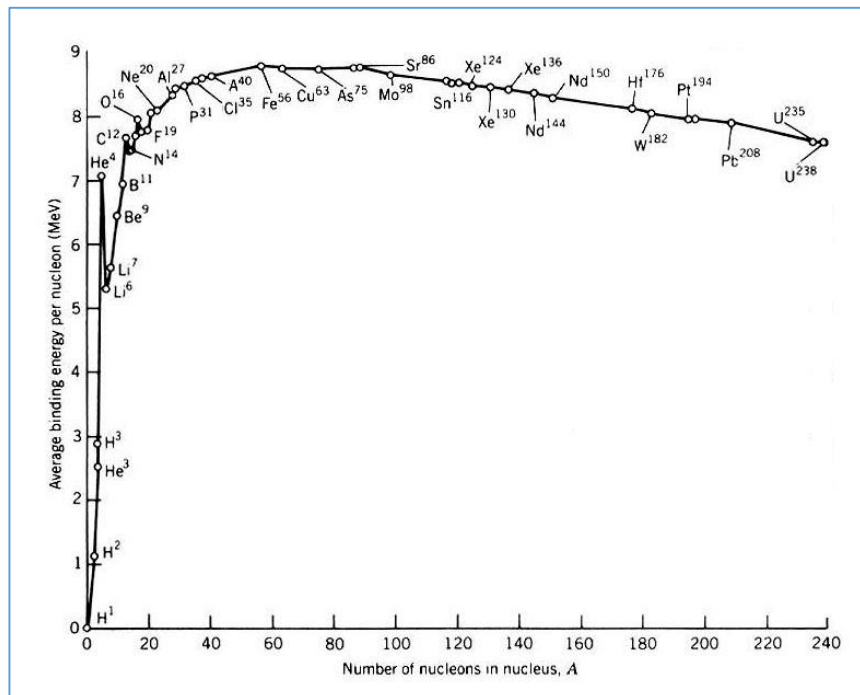
This mass difference can be converted into energy with Einstein's formula

$$\Delta E = mc^2 = 0.158829 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.43 \times 10^{-11} \text{ J}$$
$$\text{or } \frac{1.43 \times 10^{-11}}{1.6 \times 10^{-19}} = 8.93 \times 10^7 \text{ eV}$$

The binding energy is actually an **energy deficit** of the nucleus; it is the energy that must be applied to pull the nucleus apart.

To compare different nuclei we must divide the binding energy of the nucleus by the number of nucleons. This **binding energy per nucleon** can be depicted in a graph against mass number.

*Binding Energy graph.
Image source:*



www.commonswikimedia.org

The element with the largest binding energy per nucleon is $^{56}_{26}\text{Fe}$ (Iron). This is therefore the most stable element. The lighter elements are gradually less stable and the heavier elements also become a little less stable towards high mass numbers. Note that ^4_2He (Helium) is in an anomalous position in the graph; it is more stable than expected for its mass number. $^{12}_6\text{C}$ (Carbon) and $^{16}_8\text{O}$ (Oxygen) are also slightly outside the curve and thus more stable than their mass number suggests.

Fission

When a massive nucleus splits into smaller components, we can see in the graph that each of these smaller components has a larger binding energy per nucleon than the original massive nucleus. This mass difference will be released as energy when the heavy nucleus splits. This is energy released from **nuclear fission** of heavy elements.

Fusion

In a similar way, when two light nuclei fuse together to form a heavier nucleus, the final binding energy per nucleon is larger than the original value. This means that energy is released from **nuclear fusion** of light elements.

WORKED EXAMPLES

(Mass in kg)

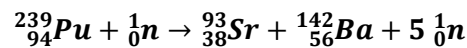
Calculate the Binding Energy per Nucleon for ${}^4_2\text{He}$.

The mass of a Helium nucleus is		6.644661x10 ⁻²⁷
The mass of two protons	2 x 1.672623x10 ⁻²⁷ =	3.345246x10 ⁻²⁷
and of two neutrons	2 x 1.674929x10 ⁻²⁷ =	3.349858x10 ⁻²⁷
Total mass of nucleons		6.695104 x10 ⁻²⁷
Mass deficit is	6.695104 x10 ⁻²⁷ - 6.644661x10 ⁻²⁷ =	0.050443x10 ⁻²⁷
Binding Energy ($\Delta E = mc^2$)		4.539870x10 ⁻¹² J
or		2.83x10 ⁷ eV
Binding energy per nucleon is	2.83x10 ⁷ / 4 =	7.08x10 ⁶ eV

or **7.08 MeV** (see graph)

Energy from Fission

Plutonium splits into Strontium and Barium and releases 5 neutrons:



Rest mass before:	${}^{239}_{94}\text{Pu}$	396.92935x10 ⁻²⁷
	${}^1_0\text{n}$	1.67483x10 ⁻²⁷
	Total	398.60418x10⁻²⁷

Rest mass after:	${}^{93}_{38}\text{Sr}$	154.27837x10 ⁻²⁷
	${}^{142}_{56}\text{Ba}$	235.64216x10 ⁻²⁷
	5 ${}^1_0\text{n}$	8.29468x10 ⁻²⁷
	Total	398.29468x10⁻²⁷

So the mass deficit is:

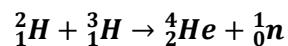
$$\Delta m = 0.3095x10^{-27}$$

Energy deficit ($\Delta E = mc^2$) is $0.3095x10^{-27} \times (2.998x10^8)^2 = 2.782x10^{-11}$ J.

One gram of Plutonium contains $\frac{1}{1000 \times 396.92935x10^{-27}} = 2.5x10^{21}$ atoms and will in this reaction produce an amount of energy of $2.5x10^{21} \times 2.782x10^{-11} = 7x10^{10}$ J or 70 GJ.

Energy from Fusion

Deuterium fuses with Tritium and this produces Helium and a neutron



Rest mass before:	${}^2_1\text{H}$	3.34330x10 ⁻²⁷
	${}^3_1\text{H}$	5.00784x10 ⁻²⁷
	Total	8.35114x10⁻²⁷

Rest mass after:	${}^4_2\text{He}$	6.64591x10 ⁻²⁷
	${}^1_0\text{n}$	1.67483x10 ⁻²⁷
	Total	8.32074x10⁻²⁷

So the mass deficit is: $\Delta m = 0.03040x10^{-27}$ which relates to $0.03040x10^{-27} \times (2.998x10^8)^2 = 0.2732 \times 10^{-11}$ J deficit in energy. This is the amount of energy produced by this reaction.