Ngā Whetū Resources

Calculus Year 13 (Level 8)

Summary 6

Conic Sections

(Use the scale diagrams below to determine all parameters of the three conic sections)

Ellipse

Distance property:

$PF_1 + PF_2 = 2a = \text{constant}$

General form: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Centre: (0,0)

a: semi-major axis

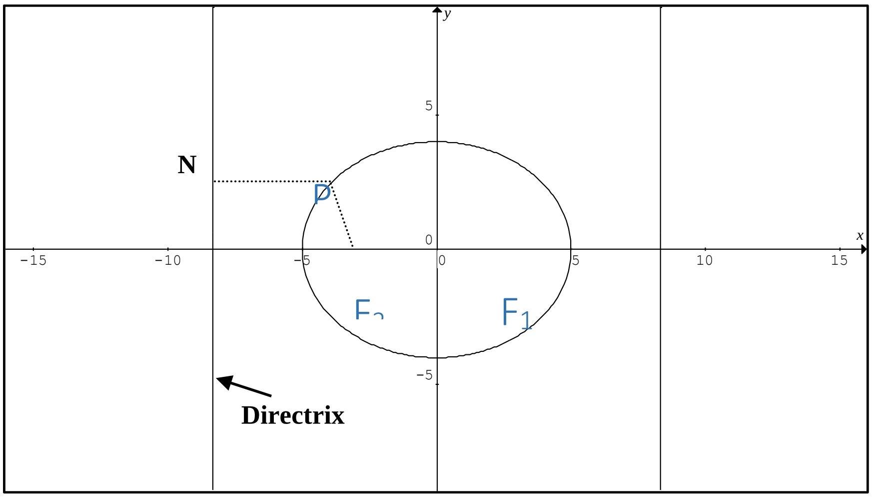
b: semi-minor axis

Eccentricity:;

$c = ae = \sqrt{a^2 - b^2}\ ; 0 < e < 1$

Foci: $(\pm ae, 0)$

Directrix: $x = \pm\dfrac{a}{e}$



Hyperbola

Distance property: $PF_1 - PF_2 = 2a = \pm\text{constant}$

General form: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Centre: (0,0)

2a: transverse axis

Eccentricity:

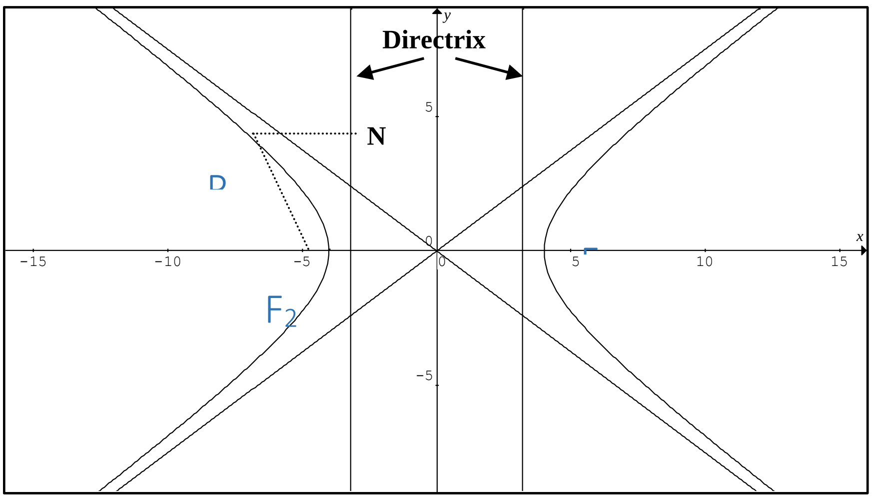
$c = ae = \sqrt{a^2 + b^2}\ ;\ e > 1$

Vertices: $(\pm a, 0)$

Foci: $(\pm ae, 0)$

Asymptotes: $y = \pm\dfrac{b}{a}x$

Directrix: $x = \pm\dfrac{a}{e}$



Rectangular Hyperbola: $a \equiv b$ hence

Asymptote: $y = \pm x$ and $e^2 - 1 = \dfrac{a^2}{b^2} = 1$ or $e^2 = 2$ and $e = \sqrt{2}$

Parabola

Distance property: $\boxed{PF = PN}$

General form: $y^2 = 4ax$

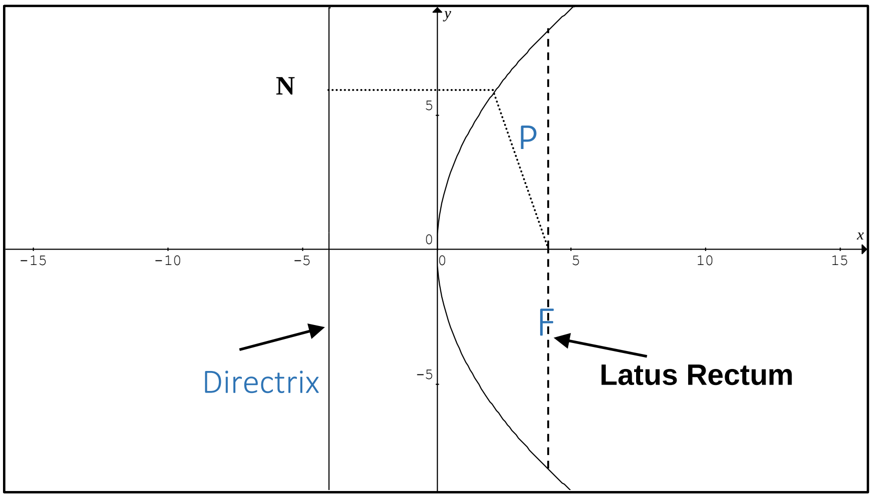
focal length: **a**

latus rectum: **4a**

Focus: $(a, 0)$

Eccentricity: $\dfrac{PF}{PN} = e = 1$

Directrix: $x = -a$



Note: *If Conic Section is translated over* (x_1, y_1) *(general form e.g.* $\dfrac{(x-x_1)^2}{a^2} - \dfrac{(y-y_1)^2}{b^2}$*) then add* (x_1, y_1) *to all calculated co-ordinates (foci, vertices, etc.) and replace x and y in equations (for asymptote, directrix, etc.) by* $(x-x_1)$ *and* $(y-y_1)$ *resp.*

Example: Hyperbola $\dfrac{(x-3)^2}{9^2} - \dfrac{(y-2)^2}{4^2} = 1$. *Centre:* $(3, 2)$ *; vertices:*

$(\pm a + 3, 0 + 2) = (6, 2)$ *and* $(0, 2)$ *Asymptotes:* $(y-2) = \pm \dfrac{b}{a}(x-3)$ *etc.*

Focus-Directrix property

Ellipse	$\dfrac{PF}{PN} = e < 1$
Hyperbola	$\dfrac{PF}{PN} = e > 1$
Parabola	$\dfrac{PF}{PN} = 1$

Tangents and Intersections

Find intersection of straight line and conic section by solving two simultaneous equations resulting in **quadratic equation** (general form: $ax^2 + bx + c = 0$

A straight line intersects a conic section :	Discriminant of the quadratic equation $\Delta = b^2 - 4ac$
at two points	$\Delta > 0$
at one point (tangent)	$\Delta = 0$
not at all	$\Delta < 0$

Tangents and Normals to Conics at point (x_1, y_1)

"Half-replace" x and y by x_1 and y_1 :

Example: **Tangent** at curve $x^2 + y^2 + 4x + 2y - 20 = 0$ in point $(1,3)$:

$x \times 1 + y \times 3 + 2(x+1) + (y+3) - 20 = 0$ or $3x + 4y - 15 = 0$ or $y = -\dfrac{3}{4}x + \dfrac{15}{4}$

Gradient of the **Normal** is (inverse reciprocal) $+\dfrac{4}{3}$ hence the normal at this point is

$y - 3 = \dfrac{4}{3}(x-1)$ or $y = \dfrac{4}{3}x + \dfrac{5}{3}$